# Determination of Microwave Two-Port Noise Parameters Through Computer-Aided Frequency-Conversion Techniques

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***Abstract*—A method to determine noise parameters of microwave linear two-ports (transistors) is presented, which is based on a two-channel noise temperature measuring system and an analytical data processing procedure. As compared with the one-channel measurements and the graphical processing techniques, the method offers advantages from both accuracy and experiment viewpoints. Experimental verifications related to noise parameters determination for a microwave transistor as function of frequency in *S* band are reported.**

## I. Introduction

THE DETERMINATION of the spot noise parameters of a linear two-port requires some measurements of effective noise temperatures or noise figures for different values of the source admittance, i.e., of the admittance of the two-port input termination [1]. The (redundant) experimental data so obtained are then processed through graphical [1], [2] or analytical [3] procedures.

Consequently the experimenters are faced with two main problems, namely performing measurements with good accuracy and processing data in a proper manner. On these subjects two papers appeared recently in this TRANSACTIONS. In the first paper [4] a frequency-conversion measuring technique is analyzed in which the noise signals at image frequency arising from the converter are not suppressed by a filter. In other words, it is a two-channel system (like a radiometer) for which it is shown that image frequency effects on noise-temperature measurements versus source admittance can be accounted for through analytical expressions. Since the relationships depend on the same noise parameters which have to be determined, a calculation through a successive approximation procedure is suggested.

The second paper [5] regards a computer-aided technique for processing the data obtained by means of a one-channel measuring system, i.e., with image-frequency filtering. Useful suggestions are also given to avoid the serious inconveniences already observed in processing data [3] by means of the least-squares fitting procedure which the technique is based on.

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In this paper, a method for the determination of microwave noise parameters of linear two-ports is presented, which can be seen as an application of the theoretical conclusions already derived for the two-channel measuring system, and as an improvement of the aforementioned processing technique which is rearranged here to process experimental data as yielded by measurements performed with a two-channel system.

Since the processing procedure is now able to derive directly from two-channel measurements the microwave noise parameters at the frequency of interest, any correction of data from image-frequency effects before processing them is avoided. In addition, if required, the noise parameters at the image frequency can be determined. Other advantages are pointed out in the concluding remarks.

Although the method is described with reference to a set of noise parameters more useful at microwaves, the results can be easily extended to cases in which other noise parameters are involved.

As experimental verifications the four spot noise parameters of a microwave transistor as functions of frequency in *S* band are reported.

## II. Description of the Method

The simplified block diagram of a two-channel noise-temperature measuring system which employs a frequency converter and an intermediate-frequency *Y*-factor meter is represented in Fig. 1, where the following is true.

1) $T_1$ and $T_2$ denote the absolute temperatures (available noise powers) of the *cold* and *hot* noise source, respectively.

2) $\Gamma_s$ and $\Gamma_1$ are reflection coefficients. Different values of $\Gamma_s$ are obtained by adjusting the admittance transformer network which is supposed lossless for the theoretical considerations of this section.

3) The matching network is represented as optional. It may be useful to increase the noise levels by maximizing the power gain of the two-port under test and to make $\Gamma_1 = 0$ at the frequency of interest $f$.

4) $T_e(\Gamma_s)$, $T_{e_1}(\Gamma_1)$, and $T_{e2}(\Gamma_2)$ are input effective noise temperatures, and $\alpha(\Gamma_s)$, $\alpha_1(\Gamma_1)$ ($<1$), and $\alpha_2(\Gamma_2)$ are available power gains at frequency $f$.

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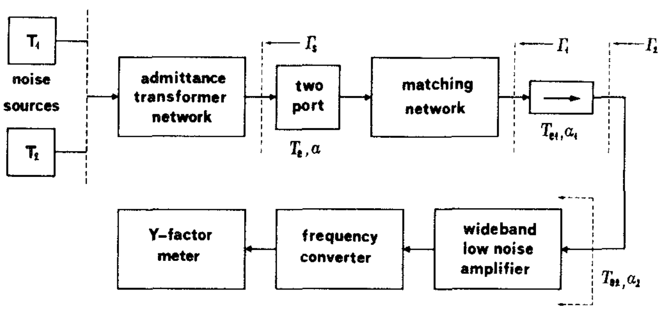


Fig. 1. Simplified block diagram of a two-channel noise temperature measuring system.

5) The isolator is useful to match (to 50 Ω) the amplifier input at both frequency of interest and image frequency $f_i = f + 2f_I$, where $f_I$ is the intermediate frequency (usually 30 MHz). In this way $\Gamma_2 = 0$ and the device under test can be regarded as an unilateral one; consequently the measurements of $T_{e2}$ and $\alpha_2$ at both frequencies for every value of $\Gamma_s$ becomes unnecessary, even though a matching network is not employed because a wide-band matching (>60 MHz) may be very difficult to realize.

Introducing now some simplifying hypotheses usually verified in practice, without losing generality, the effective noise temperature $T_e'$ of the device as derived by the meter reading, is given by (see Appendix)

$$T_e' = \frac{T_e \alpha\alpha_1 + T_{ei}\alpha_i\alpha_{1i}}{\alpha\alpha_1 + \alpha_i\alpha_{1i}} = T_{em} - \frac{T_a(2-\alpha_1-\alpha_{1i}) + 2T_{e2}}{\alpha\alpha_1 + \alpha_i\alpha_{1i}} \tag{1}$$

where the subscript $i$ identifies the values assumed by the parameters at image frequency $f_i$ and $T_a$ is the ambient temperature. The least ratio appearing in (1) (Friis term) represents the noise contributions at $f$ and $f_i$ of the stages following the device under test.

Let us introduce the formula ($T_0 = 290$ K is the standard temperature):

$$T_e(\Gamma_s) = T_{eo} + 4T_0 N \frac{|\Gamma_s - \Gamma_o|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_o|^2)} \tag{2}$$

in the linearized form [5]:

$$T_e(\Gamma_s) = a + b\frac{1}{1-\rho_s^2} + c\frac{\rho_s}{1-\rho_s^2}\cos\theta_s + d\frac{\rho_s}{1-\rho_s^2}\sin\theta_s, \qquad \text{with } \rho_s \exp j\theta_s = \Gamma_s. \tag{3}$$

The *microwave noise parameters* $T_{eo}$, minimum noise temperature, $\Gamma_o = \rho_o \exp j\theta_o$, optimum value of $\Gamma_s$, i.e., the value of $\Gamma_s$ at which the minimum value $T_{eo}$ occurs, and $N$, the Lange's terminal invariant parameter [6], are related to the linear noise parameters $a$, $b$, $c$, and $d$ through the relationships:

$$T_{eo} = a + \frac{b+\Delta}{2} \qquad N = \frac{\Delta}{4T_0}$$

$$\rho_o = \left(\frac{b-\Delta}{b+\Delta}\right)^{1/2} \quad \theta_o = \tan^{-1}\frac{d}{c}, \quad \text{with } \Delta = (b^2 - c^2 - d^2)^{1/2}. \tag{4}$$

Expressions similar to (2), (3), and (4) hold with the subscript $i$ identifying the values assumed at image frequency $f_i$ by the effective noise temperature and the two sets of noise parameters.

Introducing in (1) the expressions of $T_e(\Gamma_s)$ and $T_{ei}(\Gamma_{si})$ according to (3), we have

$$T_e' = \frac{\alpha\alpha_1}{\alpha\alpha_1 + \alpha_i\alpha_{1i}}\left(a + b\frac{1}{1-\rho_s^2} + c\frac{\rho_s}{1-\rho_s^2}\cdot\cos\theta_s + d\frac{\rho_s}{1-\rho_s^2}\sin\theta_s\right) + \frac{\alpha_i\alpha_{1i}}{\alpha\alpha_1 + \alpha_i\alpha_{1i}}\left(a_i + b_i\frac{1}{1-\rho_{si}^2} + c_i\frac{\rho_{si}}{1-\rho_{si}^2}\cdot\cos\theta_{si} + d_i\frac{\rho_{si}}{1-\rho_{si}^2}\sin\theta_{si}\right). \tag{5}$$

Applying to this linear relationship a least-squares fitting procedure as described in the following, the two sets of parameters $a$, $b$, $c$, $d$, and $a_i$, $b_i$, $c_i$, and $d_i$ can be obtained. The microwave noise parameters of the device at $f$ and $f_i$ are then determined from (4).

However, to the end of simplifying the analytical developments, the case in which the device under test can be considered as a wide-band one (e.g., a transistor) is analyzed. In this case the conditions $a_i \simeq a$, $b_i \simeq b$, $c_i \simeq c$, and $d_i \simeq d$ hold.

Consequently (5) reduces to

$$\begin{aligned} T_e' = a + \frac{1}{\alpha\alpha_1 + \alpha_i\alpha_{1i}}\Bigg[ & b\left(\frac{\alpha\alpha_1}{1-\rho_s^2} + \frac{\alpha_i\alpha_{1i}}{1-\rho_{si}^2}\right) \\ & + c\left(\frac{\alpha\alpha_1\rho_s}{1-\rho_s^2}\cos\theta_s + \frac{\alpha_i\alpha_{1i}\rho_{si}}{1-\rho_{si}^2}\cos\theta_{si}\right) \\ & + d\left(\frac{\alpha\alpha_1\rho_s}{1-\rho_s^2}\sin\theta_s + \frac{\alpha_i\alpha_{1i}\rho_{si}}{1-\rho_{si}^2}\sin\theta_{si}\right)\Bigg] \\ = a + b\beta + c\gamma + d\delta. \end{aligned} \tag{6}$$

Let us define the error function

$$e = \frac{1}{2}\sum_{k=1}^{n}(T_{ek}' - T_{esk})^2 = \frac{1}{2}\sum_{k=1}^{n} P_k^2 = \frac{1}{2}\sum_{k=1}^{n}(a + b\beta_k + c\gamma_k + d\delta_k - T_{esk})^2 \tag{7}$$

where $k$ is the number of the sets $T_{esk}$, $\rho_{sk}$, and $\theta_{sk}$ (with $k = 1 \div n$) of the experimental data, and $T_{ek}'$ represents the corresponding values assumed by $T_e'$ given by (6). Since

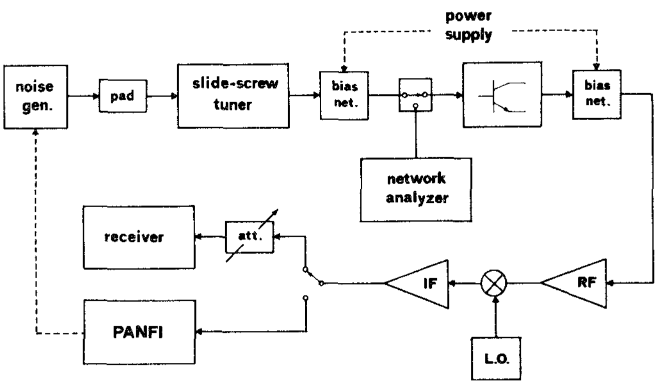


Fig. 2. Noise temperature measuring setup used for the experimental verifications.

the conditions for minimum error are

$$\frac{\delta e}{\delta a}=\sum_k P_k=0 \quad \frac{\delta e}{\delta b}=\sum_k P_k\beta_k=0$$
$$\frac{\delta e}{\delta c}=\sum_k P_k\gamma_k=0 \quad \frac{\delta e}{\delta d}=\sum_k P_k\delta_k=0 \tag{8}$$

by means of a simple computing program for linear-equation system solution, the noise parameters are determined from (8) and (4) with an increasing accuracy as the redundancy of the experimental data increases (say $n=$ 7–10).

## III. Experimental Verifications

Many experimental verifications of the described method have been carried out. The ones reported here as an example regard noise parameters determination for a microwave transistor in the range 2–4 GHz.

The noise temperature measuring setup used is shown in Fig. 2.

An on–off switched gas-discharge noise generator is employed as the noise source; for such a device the hypotheses $T_1 \simeq T_{1i}$ and $T_2 \simeq T_{2i}$ hold when the intermediate frequency is $f_I=30$ MHz; the excess noise ratio is $15.6 \pm 0.25$ dB.

The noise generator is connected to the transformer network through an attenuator (or an isolator) to reduce source mismatch effects, avoiding the need for further corrections when processing experimental data [7], [8]. However, when a switched gas generator is employed, the attenuator (3-dB minimum) is necessary to prevent damages to the solid-state device under test due to residues of high-voltage ignition spikes.

The source-admittance transformation is realized by means of a slide-screw tuner by sliding the carriage for two different penetrations of the tuning slug, obtaining values of $\Gamma_s$ (and $\Gamma_{si}$) which lie nearly on two different centered circles on the Smith chart [5]. For accuracy reasons, values of $\Gamma_s$ in the neighborhood of the optimum value $\Gamma_o$ are selected. The region in which $\Gamma_o$ will lie approximately is determined with the aid of a precision automatic noise figure indicator (PANFI). When performing the other measurements as function of $\Gamma_s$, more accuracy is obtained by substituting this meter with a precision test receiver and a variable attenuator ($Y$-factor method).

When necessary, the frequency-dependent insertion losses of the admittance transformer network are evaluated and accounted for. The transistor under test is allocated into a jig connected to input and output biasing network.

Between the slide-screw tuner and the input bias network a (remote driven) SPDT microwave switch is inserted, which allows one to measure on-line $\Gamma_s$ and $\Gamma_{si}$ by means of a network analyzer.

Since the parameters $T_{eo}$ (or the noise figure $F_o$), $\rho_o$, and $N$ are terminal invariant under lossless transformation, after data processing only the parameter $\theta_o$ is to be corrected for the electrical length between the measuring plane and the transistor input.

Because of the difficulty of matching in a 60-MHz bandwidth, the matching network is not employed, and the available power gains ($<1$) $\alpha_1$ and $\alpha_{1i}$ of the isolator are computed as a function of $\Gamma_1 \simeq \Gamma_{1i}$.

After measuring the noise temperature $T_{e2}$ of the RF amplifier-mixer-IF amplifier-meter cascade, the value of $T_e'$ can be obtained from (1), if the transistor available power gains $\alpha(\Gamma_s)$ and $\alpha_i(\Gamma_{si})$ (i.e., scattering parameters) are known.

From (8), (6), and (4) the noise parameters are then determined through data processing. In our case the IBM subroutine SIM Q translated in Basic language is used on a desk computer.

The values of the noise parameters of a transistor AT-4642 (AVANTEK-CE configuration; $I_C=5$ mA, $V_{CE}=10$ V) in the frequency range 2–4 GHz are reported in Fig. 3.

In order to prove the validity of the method, twenty measurements of noise temperature are carried out at a given frequency and only $n=10$ of them are processed, verifying that the results vary only slightly as different data are selected.

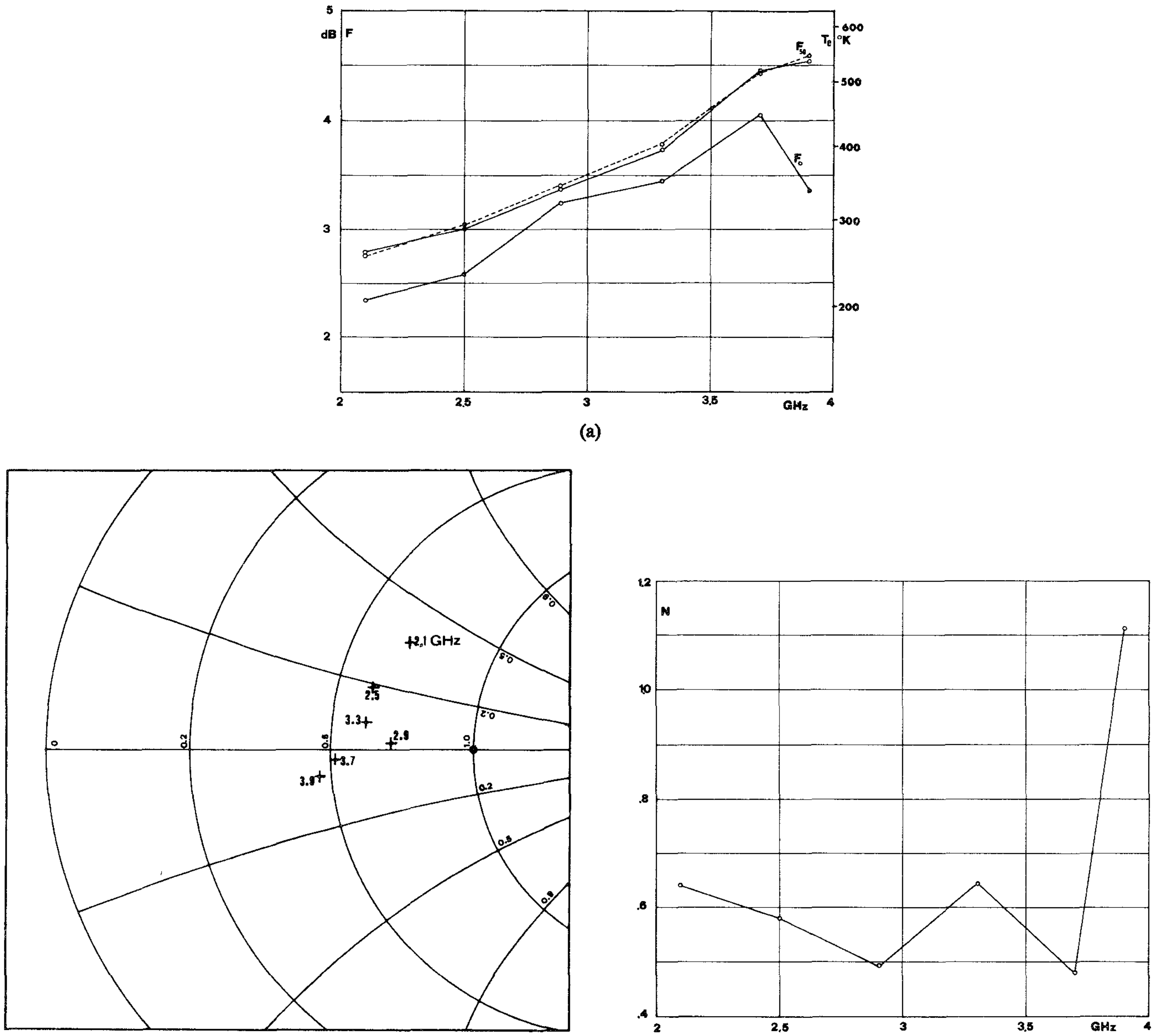


Fig. 3. Noise parameters of a transistor AT-4642 (AVANTEK—common emitter configuration; $I_C = 5$ mA, $V_{CE} = 10$ V) in the frequency range 2–4 GHz. (a) The optimum noise figure $F_o$ is reported together with the measured (----) and the computed (——) noise figure $F_{50}$ of the device in input matched condition. (b) The optimum source reflection coefficient. (c) The terminal invariant parameter $N$.

In addition, the noise temperature $T_{50}$ (or noise figure $F_{50}$) of the device in matched-input condition (50 Ω) as a function of frequency has been determined in two ways, i.e., 1) by computing $T_{50}$ by means of the noise parameters obtained, and 2) by measuring $T_{50}$ directly, noting that when $\Gamma_s$ is real the two-channel measuring system behaves as a single channel one and no corrective action is necessary to account for image frequency effects [4]. As shown in Fig. 3(a), the values of $T_{50}$ obtained as above are in very good agreement with each other.

## IV. Concluding Remarks

A method to determine microwave noise parameters of two-ports (transistors) is presented which is based on a two-channel noise temperature measuring system and an analytical data processing procedure (least-squares method).

As compared with the one-channel measurements and the graphical processing techniques, the method offers advantages from both the accuracy and the experimental viewpoint. Some of them are as follows.

Since the filter is not employed, carrying-out measurements are more simple because a careful and stable alignment of the setup through a test oscillator is not required before performing every set of measurements at a given frequency.

The method does not require careful selection of the values of source-reflection coefficients, in contrast with methods requiring that source admittances (reflectances) be carefully adjusted with either the real part (amplitude) or imaginary part (phase) exactly constant.

Accuracy is improved, because the increase of the mixer noise due to filter mismatch at image frequency is avoided.

The method permits correction of experimental data before processing for the noise of the stages following the device under test and for the noise due to the transformer network insertion loss. This leads to a better accuracy in determining the optimum noise parameters $T_{eo}$ $(F_o)$, $\rho_o$, and $\theta_o$. In addition the method permits the determination of the noise parameters of the device at both frequency of interest and image frequency, and accounts for the measuring errors due to generator mismatch effects, if any.

Many computer-simulated and experimental verifications of the method have been carried out. The experimental results reported here regard a microwave transistor tested over the range of 2–4 GHz.

## Appendix

The factor $Y$ as ratio of the noise powers measured by the meter when the two noise sources are alternatively connected to the measuring system is given by

$$Y=\frac{(T_2+T_e(\Gamma_s))\alpha(\Gamma_s)\alpha_1(\Gamma_1)+T_{e1}(\Gamma_1)+2T_{e2}+(T_2+T_{ei}(\Gamma_{si}))\alpha_i(\Gamma_{si})\alpha_{1i}(\Gamma_{1i})+T_{e1i}(\Gamma_{1i})}{(T_1+T_e(\Gamma_s))\alpha(\Gamma_s)\alpha_1(\Gamma_1)+T_{e1}(\Gamma_1)+2T_{e2}+(T_1+T_{ei}(\Gamma_{si}))\alpha_i(\Gamma_{si})\alpha_{1i}(\Gamma_{1i})+T_{e1i}(\Gamma_{1i})} \tag{A1}$$

which obviously can be derived as particular case of the more general relationship reported in [4].[1] The only difference is that (A1) refers to Fig. 1 in which the isolator realizes a wide-band (>60 MHz) matching of the microwave amplifier input ($T_{e2}=T_{e2i}$, $\alpha_2=\alpha_{2i}$), and that the hypotheses $T_1\simeq T_{1i}$ and $T_2\simeq T_{2i}$ hold.

Denoting with

$$T_{em}=\frac{T_2-YT_1}{Y-1} \tag{A2}$$

the meter reading in terms of noise temperature, and by recalling that the output effective noise temperatures of the isolator are $T_{e1}=T_a(1-\alpha_1)$ and $T_{e1i}=T_a(1-\alpha_{1i})$, where $T_a$ is the ambient temperature, from (A1) we have

$$T_{em}=\frac{T_e\alpha\alpha_1+T_{ei}\alpha_i\alpha_{1i}}{\alpha\alpha_1+\alpha_i\alpha_{1i}}+\frac{T_a(2-\alpha_1-\alpha_{1i})+2T_{e2}}{\alpha\alpha_1+\alpha_i\alpha_{1i}} \tag{A3}$$

and (1) can be derived.

[1] Unfortunately the $Y$-factor relationship in (4) contains a mistake. It should be read as $[(\cdots)\mu_2+(\cdots)\mu_{2i}]/[(\cdots)\mu_2+(\cdots)\mu_{2i}]$ instead of $[(\cdots)\mu_2/(\cdots)\mu_2]+[(\cdots)\mu_{2i}/(\cdots)\mu_{2i}]$. All the relationships derived from it are correct, however.

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